PHYSICS OF MATERIALS

Physics School Autumn 2025

Series 6 Solution **31 October 2025**

Exercise 1: Density of dislocations, Frank lattice

Considering a uniform distribution of the dislocations in three dimensions, we suppose that each dislocation stops on the free surface of the crystal, a cell of dimension a (average distance between etch-pits).

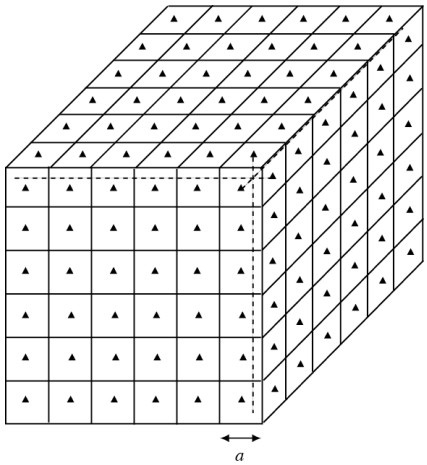

Fig. 6.1 Scheme of the etch pits appearing on each face of the cube

Each cell contains a dislocation length of $3a$, and the dislocation density is given by:

$$\Lambda = \frac{3a}{a^3} = \frac{3}{a^2}$$

Application:

$$a = 10\mu m \rightarrow \Lambda = \frac{3}{a^2} = 3{\cdot}10^6\, cm^{-2}$$

Two planes contain the Burgers vector: $b = \frac{1}{2}(\bar{1}10)$ and the planes (111) and $(\bar{1}\bar{1}1)$

Therefore, the active dislocation density averaged for "2" glide planes is: $\Lambda_m = \frac{3}{2}{\cdot}10^6\, cm^{-2}$

The Orowan equation is:

$$\varepsilon_{max} = \Lambda_m b u_{max}$$

The aluminum structure is f.c.c. The Burgers vector joins two atoms on a dense plane. Supposing that the atomic spheres are tangent in the densely packed directions:

$$b = 2R_{A1} = 2 \cdot 1.43\ \text{Å} = 2.86\ \text{Å}$$

We can also calculate b from the cell parameter: $a_{A1} = 4.05\ \text{Å} \quad b = \frac{\sqrt{2}}{2} a = 2.864\ \text{Å}$

$$\varepsilon_{max} = \frac{3}{2} \cdot 10^6 \cdot 2.86 \left[\frac{1}{cm^2}\right] \cdot 10^{-8} \cdot 1 \left[cm^2\right] = 4.3 \cdot 10^{-2} \approx 4\%$$

Exercise 2: Nabarro-Herring creep

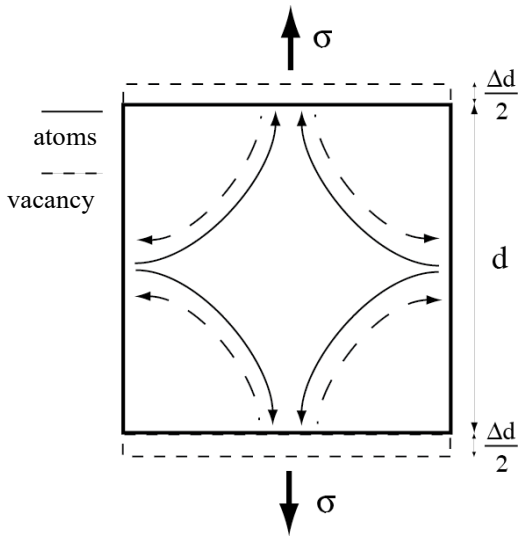

Fig. 6.2 Flux of vacancies and atoms in a grain creeping by internal diffusion

The stress σ generates a flux of atoms toward the surface and vacancies in the opposite direction. The number of atoms brought to the surface per unit of time is:

$$\phi = Jd^2 \qquad (6.2.1)$$

a) Calculation of the deformation

If the generated volume per time unit $\phi \cdot \Omega$ (Ω = atomic volume) spreads immediately, it forms on each side of the cube a layer of thickness $\frac{\Delta d}{2} = \frac{\phi \Omega t}{d^2}$. The deformation rate can be expressed as follows:

$$\dot{\varepsilon} = \frac{\Delta d}{t}\frac{1}{d} = \frac{1}{d} 2\phi \frac{\Omega}{d^2} = 2\phi \frac{\Omega}{d^3} = 2J\frac{\Omega}{d} \qquad (6.2.2)$$

b) Calculation of the atom flux J:

$$J = v_V \, C_V \qquad C_V = \frac{number\ of\ vacancies}{volume\ crystal} = \frac{n_V}{n\Omega} = \frac{X_V}{\Omega} \tag{6.2.3}$$

To calculate v_V, we consider that the stress causes a transport force F on the vacancies (Einstein formula).

$$v_V = \frac{D_{mV} F}{kT} \tag{6.2.4}$$

$D_{m\ell}$ is the diffusion coefficient per defect migration.

c) Forces

If the crystal is deformed to a length Δd and the vacancies are transported on each side by the force F over a characteristic distance $\approx \frac{d}{2}$, the energy balance gives:

$$\sigma \cdot d^2 \cdot \frac{\Delta d}{2} = F \cdot \frac{d}{2} \cdot \text{ nb vacancies} = F \cdot \frac{d}{2} \cdot \frac{(\Delta d/2) \cdot d^2}{\Omega}$$

From which we get: $F = \frac{2\sigma\Omega}{d}$ (6.2.5)

Using (6.2.3), (6.2.4) and (6.2.5) we can determine the flux J:

$$J = \frac{X_V}{\Omega} \frac{D_{mV}}{kT} \frac{2\sigma\Omega}{d} = \frac{D_{sd}}{kT} \frac{2\sigma}{d} \tag{6.2.6}$$

We note that $X_V D_{mV} = D_{sd}$ is the self-diffusion coefficient (§ 5.3.2).

Finally (6.2.1) and (6.2.2) imply: $\dot{\varepsilon} = \frac{4 D_{sd} \ \sigma \ \Omega}{d^2 \ kT}$ (6.2.7)

The creep is Newtonian ($\dot{\varepsilon} \sim \sigma$) and depends on the grain size d, varying with $\frac{1}{d^2}$, which can be experimentally verified.